Aerodynamic Features of the Flap-Balanced Swivel Airfoil

J. C. Gibbings*
University of Liverpool, Liverpool, England

The response of a lifting airfoil that is pivoted about a spanwise axis can be made stable to incidence changes by linking its movement to that of an attached flap control. The enhanced lift-curve slope obtainable, the response to incidence change, and the application of control forces and movements are considered.

Nomenclature

a_0, a_1, a_2	=	coefficients, Eqs. (1) and (2)
b	=	positive quadratic root
b_0,b_1,b_2	=	coefficients, Eqs. (5) and (6)
c	=	airfoil chord
c_{fixed}	=	chord of corresponding, fixed airfoil
C_H, C_{Hf}	=	moment coefficient about the main hinge and flap hinge, respectively
C_L	=	lift coefficient
C_m	=	pitching-moment coefficient
C_0, C_1, C_2	=	coefficients, Eq. (11)
d	=	negative quadratic root
E	=	chord of flap as a proportion of airfoil chord
f	=	floating ratio, Eq. (15)
H, H_f	=	main-hinge moment per unit span and flap-hinge moment per unit span, respectively
I_f, I_{fl}	=	moment of inertia of flap about the main hinge and about the flap hinge, respectively
I_s	=	moment of inertia of airfoil
l	=	airfoil span
L	=	lift per unit span
m	=	mass per unit area of airfoil
M	=	airfoil pitching moment
m_0, m_1, m_2	=	coefficients, Eqs. (3) and (4)
n	=	gearing ratio, Eq. (8)
P	$\equiv$	$(E^2b_1 + m_2 + \lambda a_2)/E^2b_2$
Q	$\equiv$	$(m_1 + \lambda a_1)/E^2b_2$
r	$\equiv$	$1 + 2P/[P^2 - 4Q]^{1/2}$
R	$\equiv$	$\dfrac{1}{2}\dfrac{E^2b_1}{m_1 + \lambda a_1 + E^2b_1} + \dfrac{1}{2}\dfrac{1+r}{1+b}$
V	=	velocity of flow
α, α_f, α_s	=	angle of flow to frame, flap to airfoil, and airfoil surface to frame, respectively
$\ddot{\alpha}_f$, $\ddot{\alpha}_s$	=	angular acceleration of flap and airfoil surface, respectively
λ	=	distance from airfoil neutral point to main hinge, as a fraction of the chord
ρ	=	air density
$\Delta(dC_L/d\alpha)$	=	lift-slope increment due to floating

1. Introduction

Airfoil surfaces that can be rotated about a hinge line aligned in the spanwise direction have several uses. They are, for example, used for both the horizontal and vertical tail surfaces of aircraft, for ship hydrofoil stabilizers, for ship rudders, and for submarine controls.

A surface with a symmetrical airfoil shape, being freely hinged in front of the neutral point, will align itself with the flow direction. When the hinge position is behind the neutral point, the airfoil surface will be unstable when aligned with the flow; so, the slightest disturbance will enable the aerodynamic forces to increase the angle of incidence to beyond the stalling point.

If an airfoil, hinged behind the neutral point, has a hinged-flap portion that is linked to the basic frame, so that a positive increment in the aerofoil incidence results in a positive increment in the flap angle, then this can form a stable arrangement. This arrangement is sketched in Fig. 1 where one possible form of linkage is shown; the member denoted *a-b* is fixed to the basicframe, the member *c-d* is fixed to the flap, and they are joined by the link *b-c*. The airfoil is hinged to the basic frame at *a*, and the flap is hinged to the airfoil at *d*. Thus, when the airfoil surface rotates through an angle α_s the flap rotates through an angle α_f. The present paper investigates the advantages and the limitations of this type of control surface and presents an analysis of the various ways of applying control movement.

2. The Aerodynamic Forces

The lift coefficient C_L is written as a function of the angle of the direction of the undisturbed flow α, of the airfoil surface α_s, and of the flap angle α_f, all three angles being illustrated in Fig. 1. Thus,

$$dC_L = a_1 d(\alpha + \alpha_s) + a_2\, d\alpha_f \tag{1}$$

where, as usual,

$$a_1 \equiv \partial C_L/\partial(\alpha + \alpha_s) \qquad a_2 \equiv \partial C_L/\partial \alpha_f$$

When a_1 and a_2 can be approximated by constant values, then, for an airfoil of chord c, having a lift per unit span L,

$$L/\tfrac{1}{2}\rho V^2 c \equiv C_L = a_0 + a_1(\alpha + \alpha_s) + a_2\alpha_f \tag{2}$$

Similarly, the coefficient of the airfoil pitching moment about the neutral point, C_m, is written

$$dC_m = m_1 d(\alpha + \alpha_s) + m_2 d\alpha_f \tag{3}$$

where

$$m_1 \equiv \partial C_m/\partial(\alpha + \alpha_s) \qquad \text{and} \qquad m_2 \equiv \partial C_m/\partial\alpha_f$$

so that approximating as before, for moment M per unit span,

$$M/\tfrac{1}{2}\rho V^2 c^2 \equiv C_m = m_0 + m_1(\alpha + \alpha_s) + m_2\alpha_f \tag{4}$$

The moment per unit span, applied by the flap about the

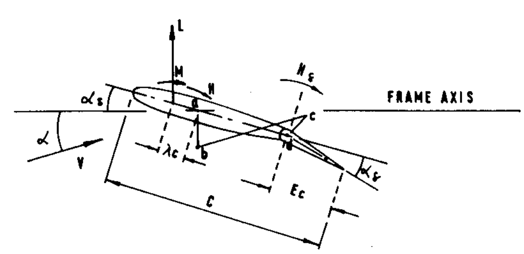

Fig. 1 Notation of flap-airfoil arrangement.

Received January 19, 1968, revision received June 28, 1968.
* Department of Mechanical Engineering, Fluid Mechanics Division.

flap hinge H_f is similarly expressed by

$$dC_{Hf} = b_1 d(\alpha + \alpha_s) + b_2 d\alpha_f \qquad (5)$$

where

$$b_1 \equiv \partial C_{Hf}/\partial(\alpha + \alpha_s) \qquad \text{and} \qquad b_2 \equiv \partial C_{Hf}/\partial \alpha_f$$

and again, for a flap chord of Ec,

$$H_f/\tfrac{1}{2}\rho V^2 E^2 c^2 \equiv C_{Hf} = b_0 + b_1(\alpha + \alpha_s) + b_2\alpha_f \qquad (6)$$

Finally, the moment per unit span, applied by the airfoil about the main hinge H, is written

$$C_H \equiv H/\tfrac{1}{2}\rho V^2 c^2 \qquad (7)$$

If the airfoil main hinge is a distance λc behind the neutral point, the work done by the aerodynamic forces when the airfoil is rotated through an angle $d\alpha_s$, the flap angle being fixed, is $Md\alpha_s + L\lambda c d\alpha_s$. The work done in further rotating the flap through an angle $d\alpha_f$ is $H_f d\alpha_f$. The sum of these two terms is equal to the work applied by the airfoil about the hinge, which is $Hd\alpha_s$. Thus,

$$Hd\alpha_s = (M + \lambda cL)d\alpha_s + H_f d\alpha_f$$

Writing the gearing ratio n as

$$n \equiv d\alpha_f/d\alpha_s \qquad (8)$$

this becomes

$$H = M + \lambda cL + nH_f$$

or

$$C_H = C_m + \lambda C_L + nE^2 C_{Hf} \qquad (9)$$

Incorporating the approximations of Eqs. (2, 4, and 6) enables this to be written as

$$C_H = C_0 + C_1(\alpha + \alpha_s) + C_2\alpha_f \qquad (10)$$

where

$$C_0 \equiv m_0 + \lambda a_0 + nE^2 b_0$$
$$C_1 \equiv m_1 + \lambda a_1 + nE^2 b_1 \qquad (11)$$
$$C_2 \equiv m_2 + \lambda a_2 + nE^2 b_2$$

Further, if n is a constant and the datum line is chosen so that α_s and α_f are zero simultaneously, then, from Eq. (8),

$$\alpha_f = n\alpha_s \qquad (12)$$

and

$$C_H + C_0 - C_1\alpha + (C_1 + nC_2)\alpha_s \qquad (13)$$

Without the approximations of Eqs. (2, 4, and 6) the differential form of Eq. (9) is

$$dC_H = dC_m + \lambda dC_L + nE^2 dC_{Hf}$$

With a substitution from Eqs. (1, 3, and 5), and noting Eq. (11), this becomes

$$dC_H = C_1 d(\alpha + \alpha_s) + C_2 d\alpha_f \qquad (14)$$

3. The Basic Design Parameters

When $C_H = 0$ the airfoil floats freely and a change in α results in a change in α_s. This is obtained by inserting Eq. (8) in Eq. (14) to give

$$0 = C_1 d\alpha + (C_1 + nC_2)d\alpha_s$$

and so defining a floating ratio f by

$$f \equiv d\alpha_s/d\alpha \qquad (15)$$

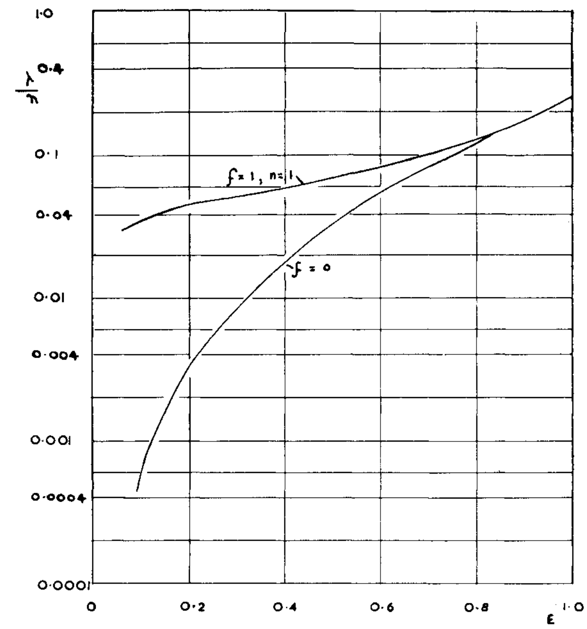

Fig. 2 Main-hinge position for a specified floating ratio as a function of flap chord.

then

$$f = -C_1/(C_1 + nC_2) \qquad (16)$$

The coefficients C_1 and C_2 are functions of λ and E; thus, there are three basic design parameters which are independent variables. They are the gear ratio n, the floating ratio f, and the flap chord E. Of these three, the first could be made a function of α_s by mechanical means.

The value of f could be decided by a requirement to avoid stalling the main airfoil surface before attainment of the maximum value of α. The value of n could be fixed by a similar requirement to avoid stalling of the flap at the maximum value of α_s. The value of E will be shown to affect several aerodynamic characteristics.

The main-hinge position is given by the value of λ. By substitution of Eqs. (11) into Eq. (16), and by noting that as M is measured about the neutral point, m_1 is zero, the value of λ is given by

$$\frac{\lambda}{n} = -\frac{b_1E^2\{1 + [nf/(1+f)](b_2/\delta_1)\} + [f/(1+f)]m_2}{a_1\{1 + [nf/(1+f)](a_2/a_1)\}} \qquad (17)$$

Thus, λ is a function of E, n, and f. When $f = 0$ there is no response of α_s to a change in α and then

$$\lambda/n = -b_1E^2/a_1 \qquad (18)$$

To provide numerical illustrations, the values of the coefficients in Eqs. (2, 4, and 6) were obtained from thin airfoil theory.[1] Estimates for real situations can be obtained using, for instance, the data of Refs. 2 and 3.

The value of λ/n from Eq. (18) is shown plotted as a function of E in Fig. 2. A practical difficulty is revealed by this case; for a flap chord of less than about 40% of the airfoil chord, it becomes difficult to estimate the airfoil neutral-point position with an accuracy sufficient to insure that λ is not seriously in error, thus, perhaps resulting in a negative value of f. The undesirability of this latter result is amplified later.

The position is greatly improved with a positive value of f. A curve for $f = 1$ and $n = 1$ is also drawn in Fig. 2. Now, for a flap chord as low as 20% of the airfoil chord, λ is 4.5% of the airfoil chord.

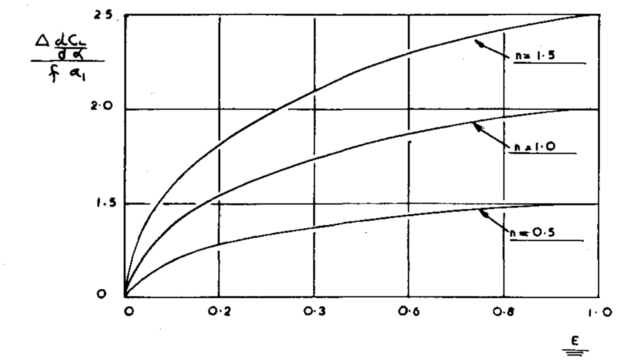

Fig. 3 Enhancement of the lift-curve slope for a specified flap-gearing ratio as a function of flap chord.

An indication of the separate effects of n and of f upon λ is given by differentiating Eq. (17) to give, respectively,

$$\left(\frac{\partial\lambda}{\partial n}\right)_{f,E} = \frac{\lambda/n - E^2(b_2/a_1)[nf/(1+f)]}{1 + [nf/(1+f)](a_2/a_1)} \tag{19}$$

and

$$\left(\frac{\partial\lambda}{\partial f}\right)_{n,E} = -\frac{E^2(b_1/a_1)(1 + nb_2/b_1) + m_2/a_1 + (1 + na_2/a_1)\lambda/n}{[(1+f)/n]\{1 + [nf/(1+f)](a_2/a_1)\}} \tag{20}$$

When $n = 1, f = 1$, and $E = 0.4$ these expressions give

$$(\partial\lambda/\partial n)_{f,E} = 0.051 \qquad \text{and} \qquad (\partial\lambda/\partial f)_{n,E} = 0.014$$

Thus, λ can be favorably increased by increases in either n or f, the former having the greater effect.

4. The Lift-Curve Slope

Substituting Eqs. (8) and (15) into Eq. (1) gives

$$dC_L/d\alpha = a_1 + f(a_1 + na_2) \tag{21}$$

With a fixed control, the lift-curve slope is a_1, thus, the fractional increment $\Delta(dC_L/d\alpha)$, under control free conditions, is

$$\frac{\Delta(dC_L/d\alpha)}{a_1} = f\left(1 + n\,\frac{a_2}{a_1}\right) \tag{22}$$

This shows that an outstanding feature of the flap-balanced airfoil is a greatly enhanced lift-curve slope under freely floating conditions, with a positive value of f and where no control force is needed.

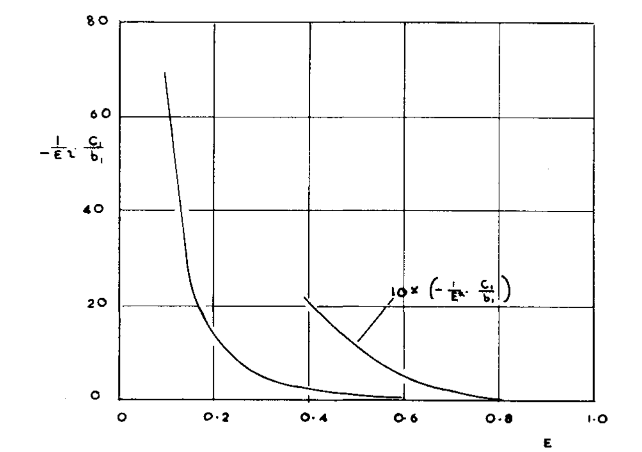

Fig. 4 Accelerating hinge moment as a function of flap chord; $f = 1.0$, $n = 1.0$.

Values are shown plotted in Fig. 3. For $f = 1$, $n = 1$, and $E = 0.4$, an increase of 173% in the lift-curve slope is seen to be available. For $f = 1.5$, $n = 1.5$, and $E = 0.6$ this increment jumps to 345%; the lift curve slope is now $4\frac{1}{2}$ times that for a fixed control.

5. Dynamic Floating Response

Under freely floating conditions, the dynamic response of the airfoil surface to a change in α is of interest. This is measured by $(\partial C_H/\partial\alpha)_{\alpha_s}$ which, by Eq. (8), implies also a constancy of α_f. From Eq. (14)

$$(\partial C_H/\partial\alpha)_{\alpha_s,\alpha_f} = C_1$$

For the ordinary flap control, from Eq. (5),

$$(\partial C_{H_f}/\partial\alpha)_{\alpha_f,\alpha_f} = b_1$$

Noting Eqs. (6) and (7), the ratio of the acceleration hinge moments, for these two cases, is

$$\frac{(\partial H/\partial\alpha)_{\alpha_s,\alpha_f}}{(\partial H_f/\partial\alpha)_{\alpha_s,\alpha_f}} = \frac{1}{E^2}\,\frac{C_1}{b_1} \tag{23}$$

Values for $f = 1$ and $n = 1$ are plotted in Fig. 4, with the right-hand portion of the graph repeated on a scale magnified 10 times. It can be seen that for flap chords of less than 52% of the airfoil chord, the acceleration-hinge moment for the flap-balanced airfoil is greater than that for the ordinary flap. The comparison is less favorable when the greater moment of inertia of the flap-balanced airfoil is taken into account.

For the ordinary flap control the angular acceleration, $\ddot{\alpha}_f$ is given by $H_f = \ddot{\alpha}_f I_{fl}$, where the moment of inertia I_{fl} is taken about the flap hinge. For the flap-balanced airfoil, the acceleration is given by $H = \ddot{\alpha}_s I_s + \ddot{\alpha}_f I_f$. Substituting from Eq. (12), this becomes $H = (I_s + nI_f)\ddot{\alpha}_s$ where I_s includes the flap and where both I_s and I_f are taken about the main hinge.

An idea of the relative order of the inertias can be obtained by regarding the airfoil and flap as thin, flat, rectangular plates of span l and of mass m per unit surface area. Then,

$$I_{fl} = \tfrac{1}{3}ml(Ec)^3$$
$$I_f = mlEc[(Ec)^2/12 + (\tfrac{3}{4} - \lambda - E/2)^2c^2]$$
$$I_s = mlc^3[\tfrac{1}{12} + (\tfrac{1}{4} - \lambda)^2]$$

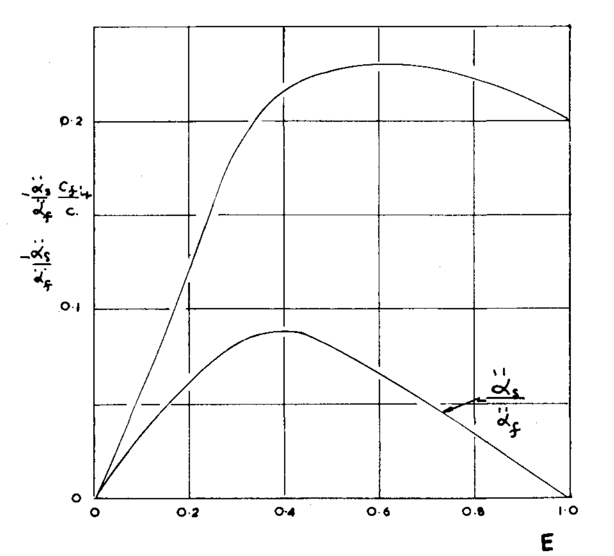

Fig. 5 Relative angular acceleration of control as a function of flap chord; $f = 1.0$, $n = 1.0$.

where the airfoil neutral point is taken as being at the quarter-chord position.

Thus, using Eq. (23),

$$\ddot{\alpha}_s/\ddot{\alpha}_f = C_1/b_1(1/E^2)[I_{f1}/(I_s + nI_f)] \qquad (24)$$

Values for $f = 1$ and $n = 1$ are shown plotted in Fig. 5, where it is seen that the maximum comparable angular acceleration of the flap-balanced control is less than one-tenth that of an ordinary flap.

This result is for a common airfoil size. If advantage is taken of the enhanced lift-curve slope of the floating flap-balanced aerofoil, the size of the latter can be reduced. For a fixed lift and aspect ratio, the airfoil chord is inversely proportional to the square root of the lift coefficient. The lift-curve slope for a fixed airfoil with an ordinary unrestrained flap is

$$\partial c_{L\,\text{fixed}}/\partial\alpha = a_1 - a_2 b_1/b_2$$

Thus, comparison with Eq. (21), gives the ratio of the chords as

$$\frac{c}{c_{\text{fixed}}} = \left[\frac{1 - (a_2/a_1)(b_1/b_2)}{1 + f(1 + na_2/a_1)}\right]^{1/2}$$

The moments of inertia being proportional to the fourth power of the chord, and the aerodynamic moments being proportional to the third power, the ratio of the accelerations is then given by the ratio of Eq. (24) times the reciprocal of the aforementioned chord ratio. Values are also plotted in Fig. 5. The maximum ratio of the acceleration is now about one fourth.

6. Application of Direct Control at the Main Hinge

With a fixed value of α, Eq. (1) shows that

$$dC_L = a_1 d\alpha_s + a_2 d\alpha_f$$

and, so, with substitution from Eq. (8),

$$(\partial\alpha_s/\partial C_L)_\alpha = 1/(a_1 + na_2) \qquad (25)$$

With an ordinary flap control, that is, with α_s fixed, $d\alpha_f/dC_L = 1/a_2$. The ratio of these two angular increments is thus

$$\frac{d\alpha_s/dC_L}{d\alpha_f/dC_L} = \frac{1}{a_1/a_2 + n}$$

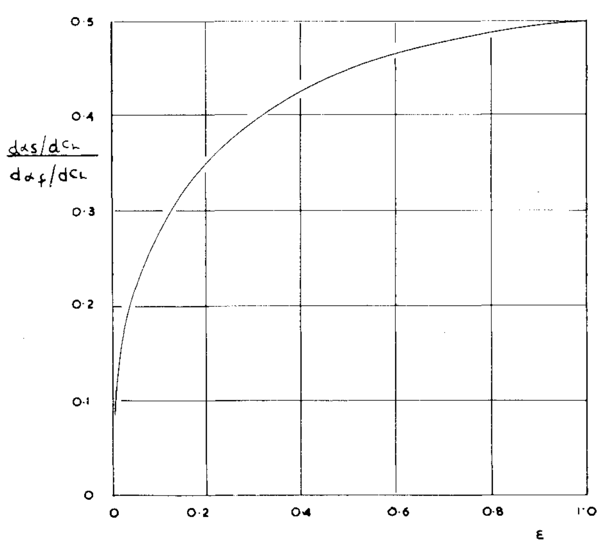

Fig. 6 Control movement per unit lift as a function of flap chord; $n = 1.0$.

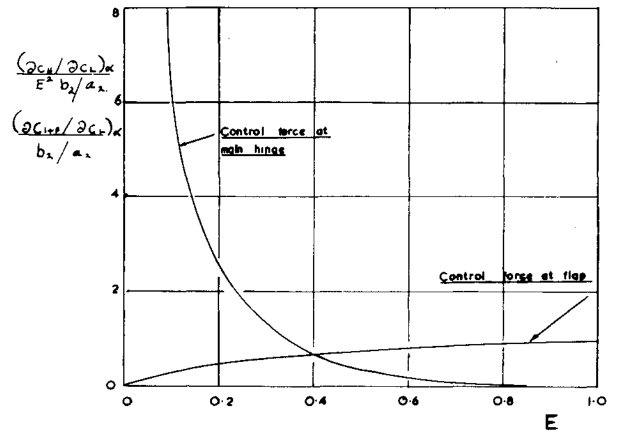

Fig. 7 Hinge moment per unit lift as a function of flap chord; $f = 1.0$, $n = 1.0$.

Values for $n = 1$ are plotted in Fig. 6, which shows that the control movement for a specified lift on the flap-balanced airfoil is under half that for the ordinary flap control.

Again, with a fixed value of α, Eq. (14) gives

$$dC_H = C_1 d\alpha_s + C_2 d\alpha_f$$

so that

$$(\partial C_H/\partial\alpha_s)_\alpha = C_1 + nC_2$$

Combining this with Eq. (25) gives the rate of change of hinge moment, with respect to lift, and due to control application, as

$$(\partial C_H/\partial C_L)_\alpha = (C_1 + nC_2)/(a_1 + na_2)$$

Using Eq. (16), this can be rewritten

$$(\partial C_H/\partial C_L)_\alpha = -(1/f)C_1/(a_1 + na_2)$$

The corresponding expression for the ordinary flap control is

$$(\partial C_{Hf}/\partial C_L)_{\alpha,\alpha_s} = b_2/a_2$$

Noting Eqs. (6) and (7) gives the ratio of the hinge moments for these two cases as

$$\frac{(\partial H/\partial C_L)_\alpha}{(\partial H_f/\partial C_L)_{\alpha,\alpha_s}} = -\frac{1}{fE^2}\frac{C_1}{b_2}\frac{1}{a_1/a_2 + n}$$

Values for $f = 1$ and $n = 1$ are plotted in Fig. 7, showing that the control force per unit lift for the flap-balanced airfoil is lower than that for the ordinary flap control when E is greater than 0.32; when $E = 0.6$ the ratio is only one fifth.

A disadvantage of this form of control is that if the airfoil is floating free then, when the control force is applied directly, the controls, and hence the applied force must move with the floating movement. This could be avoided by, for instance, applying the force via a body force that results from a uniform force field.

7. Application of Control by Variation of the Gearing Ratio

Under floating conditions, and when both α and λ are constants, a change in the gearing ratio n will result in a change in the angle of the airfoil surface, α_s. The total resulting change in the hinge moment is zero under these floating conditions, and so

$$0 = \delta C_H = (\partial C_H/\partial n)\delta n + (\partial C_H/\partial\alpha_s)\delta\alpha_s \qquad (26)$$

From Eq. (9)

$$\frac{\partial C_H}{\partial n} = \frac{\partial C_m}{\partial n} + \lambda\frac{\partial C_L}{\partial n} + E^2\left(C_{Hf} + n\frac{\partial C_{Hf}}{\partial n}\right)$$

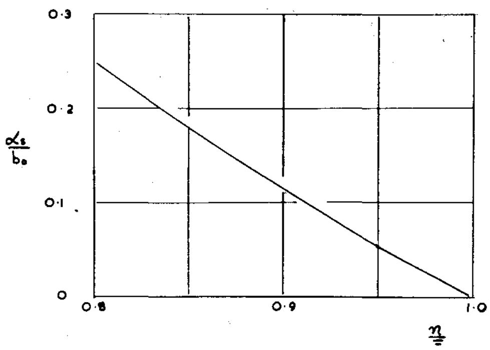

Fig. 8 Change in control angle with change in gearing ratio; $E = 0.4, f = 1.0$ at $\alpha_s = 0$.

On the right-hand side of this equation, the differentials can be expressed by means of Eq. (12) as

$$\partial/\partial n = (d\alpha_f/dn)(\partial/\partial\alpha_f) = \alpha_s\partial/\partial\alpha_f$$

Thus,

$$\partial C_H/\partial n = \alpha_s(m_2 + \lambda a_2 + nE^2b_2) + E^2C_{Hf} \quad (27)$$
$$= C_2\alpha_s + E^2C_{Hf}$$

From Eqs. (8) and (14)

$$\partial C_H/\partial\alpha_s = C_1 + nC_2$$

Substituting this relation, together with Eq. (27), into Eq. (26) gives

$$(\partial\alpha_s/\partial n)_{\alpha,\lambda} = -(C_2\alpha_s + E^2C_{Hf})/(C_1 + nC_2) \quad (28)$$

Under floating conditions Eq. (6) can be written

$$C_{Hf} = b_0 + [b_1(1/f + 1) + b_2n]\alpha_s$$

and so Eq. (28) becomes

$$\left(\frac{\partial\alpha_s}{\partial n}\right)_{\alpha,\lambda} = -\frac{E^2b_0 + [C_2 + E^2\{b_1(1+f)/f + b_2n\}]\alpha_s}{C_1 + nC_2} \quad (29)$$

A disadvantage of this type of control is now apparent in that, when $\alpha_s = 0$, control is effective only through existence of a term in b_0. However, a value of b_0 can always be introduced by, for instance, a tab control on the flap. There is the advantage that this type of control requires no control force, and a floating action is not transmitted back through the control system.

Equation (29) is a first-order, linear differential equation for α_s, the solution of which is

$$\frac{\alpha_s}{b_0} = -\frac{1}{b_2}\frac{n+d}{C_1(n+b)^r}\int\frac{C_1(n+b)^{r-1}}{(n+d)^2}dn \quad (30)$$

where

$$(n+b)(n+d) \equiv (C_1 + nC_2)/E^2b_2$$

and

$$r = 1 + 2P/(P^2 - 4Q)^{1/2}$$

in which

$$P \equiv (E^2b_1 + m_2 + \lambda a_2)/E^2b_2$$
$$Q \equiv (m_1 + \lambda a_1)/E^2b_2$$

A numerical example, for $E = 0.4$ and with both $n = 1$ and $f = 1$ at $\alpha_s = 0$, is illustrated in Fig. 8.

A limitation to the control appears when $n = -d$, for then the value of $(C_1 + n\,C_2)$ becomes zero and so, from Eq. (16), the floating ratio f becomes infinite. In this numerical example $d = -0.760$.

The variation shown in Fig. 8 is closely linear, justifying expressing the solution in series form. The result obtained is

$$\frac{\alpha_s}{b_0} = \frac{1}{b_2}\frac{n-1}{(d+1)(b+1)}[1 - R(n-1) + \ldots]$$

where

$$R \equiv \tfrac{1}{2}E^2b_1/(m_1 + \lambda a_1 + E^2b_1) + \tfrac{1}{2}(1+r)/(1+b)$$

and a plot of calculated values in Fig. 8 is found to be barely distinguishable from the exact curve given.

The increment of lift resulting from application of this control is given by

$$\delta C_L = (\partial C_L/\partial\alpha_s)\delta\alpha_s + (\partial C_L/\partial n)\delta_n$$
$$= [(\partial C_L/\partial\alpha_s)\partial\alpha_s/\partial n + \partial C_L/\partial n]\delta n$$

From Eq. (1), $\partial C_L/\partial\alpha_s = a_1 + na_2$ and, as before,

$$\partial C_L/\partial n = \alpha_s\partial C_L/\partial\alpha_f = \alpha_s a_2$$

Thus,

$$dC_L/dn = a_2\alpha_s + (a_1 + na_2)(\partial\alpha_s/\partial n)_{\alpha,\lambda}$$

the differential being given by Eq. (29) after solution by Eq. (30).

8. Application of Control by Variation of the Flap Angle

Applying an increment of flap angle, $\delta\alpha_{f0}$, results in a change of flap-hinge moment δC_{Hf}. Under floating conditions at constant α,

$$0 = \delta C_H = (\partial C_H/\partial\alpha_f)\delta\alpha_{f0} + (\partial C_H/\partial\alpha_s)\delta\alpha_s$$

With substitution from Eqs. (8) and (14), this becomes

$$0 = C_2\delta\alpha_{f0} + (C_1 + nC_2)\delta\alpha_s$$

giving

$$\partial\alpha_s/\partial\alpha_{f0} = -C_2/(C_1 + nC_2)$$

Further substitution from Eq. (16) enables this to be expressed as

$$\partial\alpha_s/\partial\alpha_{f0} = -(1+f)/n \quad (31)$$

The total change in flap angle is

$$n\delta\alpha_s + \delta\alpha_{f0} = [n - n/(1+f)]\delta\alpha_s$$
$$= [nf/(1+f)]\delta\alpha_s$$

The corresponding increment in lift is given by

$$\delta C_L = (\partial C_L/\partial\alpha_f)\delta\alpha_{f0} + (\partial C_L/\partial\alpha_s)\delta\alpha_s$$

which, with substitution from Eqs. (1) and (25), gives

$$\partial C_L/\partial\alpha_{f0} = a_2 + (a_1 + na_2)\partial\alpha_s/\partial\alpha_{f0}$$

Further substitution from Eq. (31) gives

$$\partial C_L/\partial\alpha_{f0} = a_2 - (a_1 + na_2)(1+f)/n \quad (32)$$

Similarly, the increment in flap-hinge moment is

$$\delta C_{Hf} = (\partial C_{Hf}/\partial\alpha_f)\delta\alpha_{s0} + (\partial C_{Hf}/\partial\alpha_s)\delta\alpha_s$$

and with substitution from Eqs. (5) and (31),

$$\partial C_{Hf}/\partial\alpha_{f0} = b_2 + (b_1 + nb_2)(\partial\alpha_s/\partial\alpha_{f0})$$
$$= b_2 - (b_1 + nb_2)(1+f)/n$$

Dividing this relation by Eq. (32) gives

$$\frac{\partial C_{Hf}}{\partial C_L} = \frac{b_2 - (b_1 + nb_2)(1 + f)/n}{a_2 - (a_1 + na_2)(1 + f)/n}$$

With a conventional flap control,

$$\partial C_{Hf}/\partial C_L = b_2/a_2$$

Thus, a comparison between these two types of control is given by

$$\frac{(\partial C_{Hf}/\partial C_L)_2}{b_2/a_2} = \frac{[(1 + f)/nf]b_1/b_2 + 1}{[(1 + f)/nf]a_1/a_2 + 1}$$

Values are plotted in Fig. 7 for $f = n = 1$. This shows that this control force for the floating airfoil is lower than for the conventional flap control. This type of control also has the advantage that movement due to the floating action is not transmitted back through the control system.

9. Variants For the Application of Control

Control could be obtained by a combination of any two or all three of the methods described in the previous sections. Additionally the servo-tab-control technique could be applied to rotate the flap.

The spring-control technique could also be used, for example, by applying a control force at the main hinge through a spring, the deflection of which imparts either a change in the flap angle or a change in the gearing ratio. This conceivably could prevent a control reversal because of a shift in the basic neutral point, a difficulty that was discussed in Paragraph 3.

10. Conclusions

The type of all-moving control surface discussed here is shown to be stable to incidence changes while maintaining its lift. The lift-curve slope can be greatly enhanced above the value for a simple, fixed airfoil. It is indicated that the dynamic response in terms of the rate of control-angle increase due to an incidence change is lower for this type of control than for a plain-flap control. It is shown that the control forces per increment of lift can be smaller for this type of control when control is applied at the main hinge; it can be zero when control is obtained by a change in the gearing ratio and is lower when control is obtained by a change of flap angle.

References

[1] Glauert, H., "Theoretical Relationships for an Aerofoil with Hinged Flap," R&M 1095, April 1927, Aeronautical Research Council.

[2] "Aerodynamic Data Sheets," The Royal Aeronautical Society.

[3] Gibbings, J. C., "A Correlation for the Effect of Control Deflection upon Pitching Moment," *Journal of the Royal Aeronautical Society*, Vol. 69, Nov. 1965, p. 793.